Dana Center Mathematics PATHWAYS

Reflections on the Pathway to Calculus: Removing Barriers to Calculus

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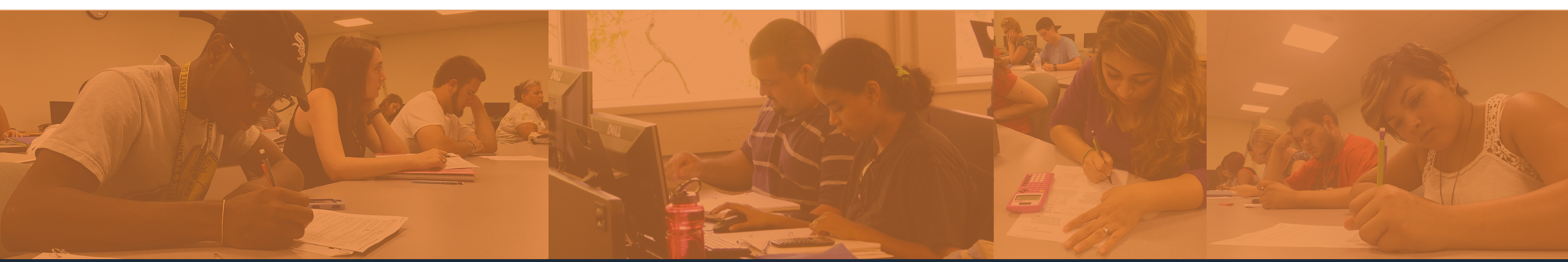


Table discussion…

- What does the pathway to calculus look like at your institution? Who does it serve? Are you having success?
- What barriers do students encounter on the pathway to calculus?

The question is…

How can we improve student outcomes?

Definition of *math pathway*

. . . a mathematics course or sequence of courses that students take to meet the requirements of their program of study.

The concept of math pathways applies to *all* students.

Dana Center Principles for Pathways

Mathematics pathways are structured so that:

1) All students, regardless of college readiness, enter directly into mathematics pathways aligned to their programs of study.

2) Students complete their first college-level math requirement in their first year of college.

Dana Center Principles for Pathways

Students engage in a high-quality learning experience in math pathways designed so that:

3) Strategies to support students as learners are integrated into courses and are aligned across the institution.

4) Instruction incorporates evidence-based curriculum and pedagogy.

An Opportunity…………

Implementation of multiple math pathways presents an opportunity to reenvision the pathway to Calculus.

Process for reenvisiong the pathway to Calculus

Content

- Utilize a back-mapping process: What are the difficult concepts/ideas students encounter in Calculus?
- Create true prerequisite courses that prepare students for Calculus instead of developing a list of favorite topics.

Architecture

- Consider the elements of instruction we know help students. For example:
  - Active and collaborative learning.
  - Meaningful experiences through contextualized mathematics.
  - Opportunities for struggle and perseverance.

Student struggle in Calculus

What are the difficult ideas/concepts students encounter in Calculus?

- Functions and function notation
- Algebra
- Concepts of inverse functions and function composition
- Communicating about change and rates of change
- Limits and approximations
- The integral as an accumulator
- Dynamic geometric reasoning
- Problem solving skills: perseverance, drawing pictures, creating equations
- Working with open-ended problem structures

Four overarching principles

1. Deep understanding of the function process (contrasted with an action view of a function)

2. Ability to apply *covariational reasoning*

3. Ability to communicate with functions and use function notation

4. Meaningful approaches to the development of algebraic reasoning

Student views of functions

Think about a precalculus course you or a colleague taught recently.

Think about how a reasonably good student might describe "functions" after leaving the course. How would they answer these two questions:

What are functions?

What are they used for?

Student views of functions

Action view of a function

- Students see functions as static.
- Computations involve evaluating a function at a single point.

Process view of a function

- Functions are processes that can be composed and inverted.
- Functions are processes that take a continuum of input values and produces a continuum of output values. i.e. Functions can be studied by examining their behavior on intervals of input values.
- Functions are used to model dynamic situations.

Reference: **Marilyn Carlson, Michael Oehrtman, and Nicole Engelke** (2010), "The precalculus concept assessment: a tool or assessing students' reasoning abilities and understandings", Cognition and Instruction, 28:2, 113-145.

Lack of developing a process view results in:

- Difficulty composing and inverting functions
- Inability to use functions effectively in word problems
- Graphs of functions are viewed as fixed curves [not as a representation of a mapping of input/output values].
- Points on graphs and slopes are viewed as [fixed] geometric properties of graphs not as properties of the underlying functions.
- Conflating the shape of a graph with the physical situation being modeled.
- Inability to interpret or express contextual relationships
- Inability to use symbols meaningfully [e.g. constructing algebraic formulas to represent relationships]

Change is central to Calculus

Think about a precalculus course you’ve recently taught…

- Do you think students left feeling that the idea of measuring and describing change was central to the course? Explain.

Covariational Reasoning

Considering how one variable changes while imagining changes in the other variable.

Five Levels of Covariational Reasoning

- Recognizing that one variable depends on another
- Identifying the direction of change of one variable as the second variable changes by a given amount
- Identifying the amount of change of one variable
- Identifying an average rate of change
- Attending to the change in the rate of change – instantaneous rate of change

These “levels” are a great help for designing content!

Reference: **Marilyn Carlson, Michael Oehrtman, and Nicole Engelke** (2010), “The precalculus concept assessment: a tool or assessing students’ reasoning abilities and understandings”, Cognition and Instruction, 28:2, 113-145.

In terms of content this means:

Explore concepts with multiple representations.

Functions are processes not algebraic formulas.

Describe behavior of functions on entire intervals.

Develop the language and inclination to describe how one quantity changes with respect to another: increasing/decreasing, rates of change, average rate of change, move towards instantaneous rate of change.

In terms of content this means:

Practice dynamical reasoning (imagine running through many input/output combinations without actually performing all of them).

Create an engaging curriculum set within authentic STEM contexts and models (exploring, creating, and interpreting mathematical models will drive the algebraic development).

Build in frequent opportunities for students to practice communicating both orally and in writing.

Make the algebra Meaningful! Give the procedures meaning!

What could this look like?

- Sample activities encompass a 25 minute learning episode
- Each activity is completed in small groups with instructor facilitation

Biological Populations

- Early in a course
- Exploring different function types, mastering use of function notation and how to communicate with functions

Note: Use of multiple representations, communication elements, average rate of change, describing change over intervals, working with covarying quantities.

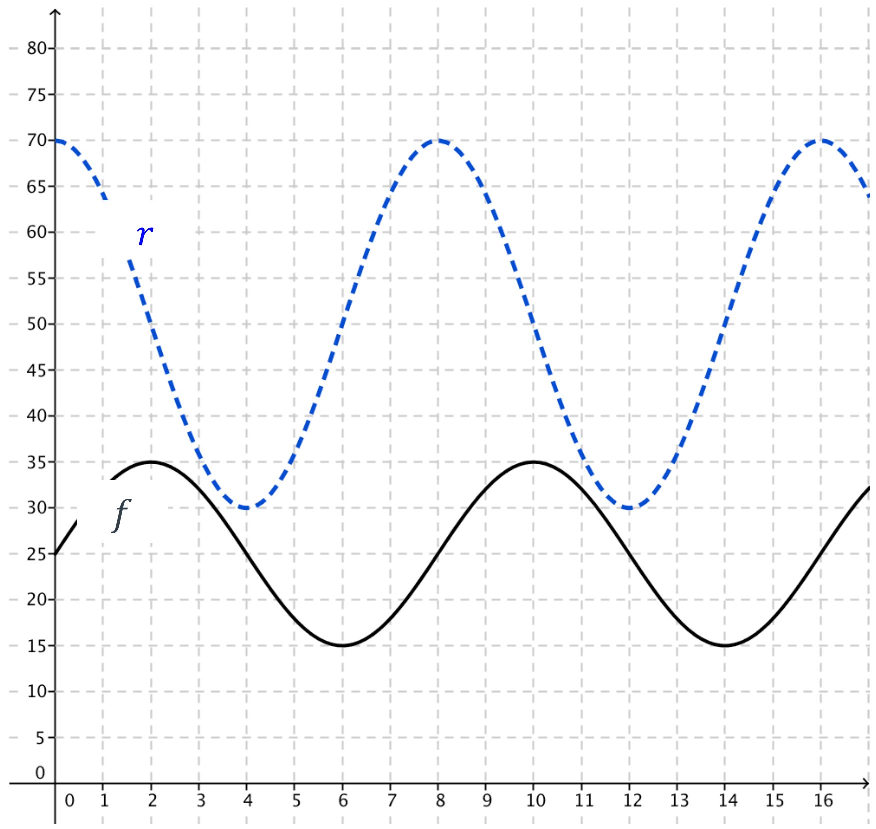
Let $r(t)$ represent the number of rabbits and $f(t)$ represent the number of foxes in a forest, where t is the number of years after 1990. The graphs of r and f appear below. Use these graphs to answer the following questions.

2) Use the graph of r to answer these questions:

Part A: Why does the function r appear to be periodic? State its period, using correct units.

Part B: Assuming the same pattern continues, what is the rabbit population in 2010?

Part C: The rabbit population is increasing between 1994 and 1998. List the next three time intervals over which the rabbit population will be increasing.



4) How are the fox and rabbit populations changing between 1990 and 1992? Why might this be the case?

5) Between 1996 and 1998, both the rabbit and fox populations are increasing.

Part A: Find the average rate of change of the rabbit population. Be sure to use correct units.

Part B: What is the average rate of change of the fox population?

6) Use the graphs to calculate the following quantities, and write sentences interpreting what each quantity represents. Use appropriate units.

Part A: $r(10) - r(7)$

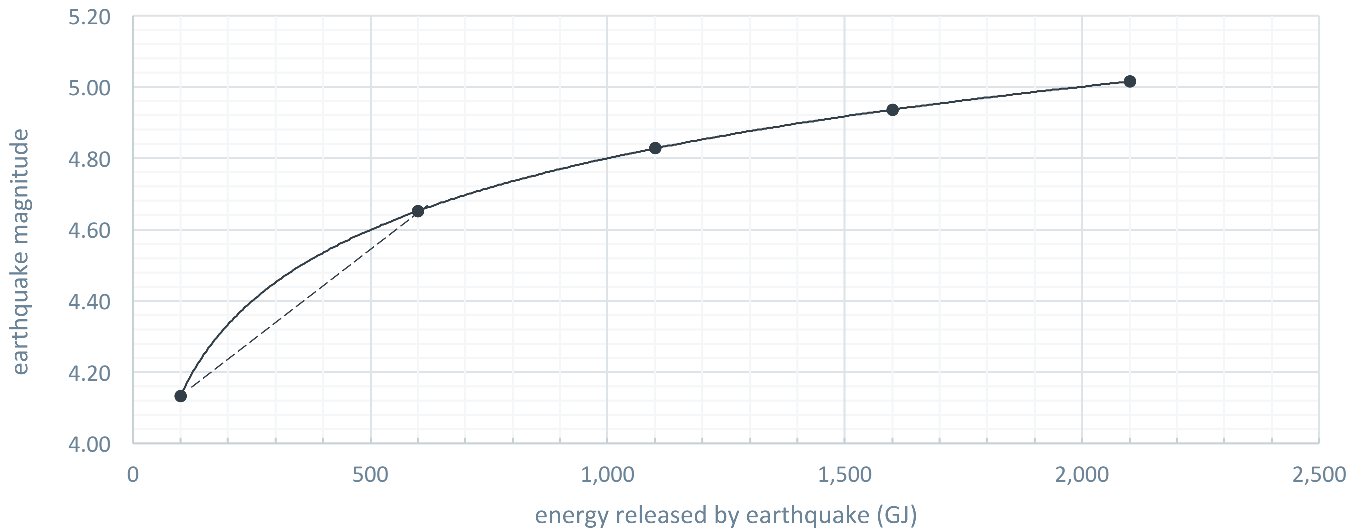
Part B: $\frac{f(6)-f(2)}{4yr}$

Geology

- Early in a course
- Exploring different function types

Note: Use of multiple representations, average rate of change connected to slope of secant line, verbal explanations of change, examining change over intervals, experiencing the rate of change of a function as a function itself.

Let's continue investigating the earthquake magnitude scale that we started in the preview assignment. Here is the data table that we were working with, along with a plot of the magnitude vs. energy released.



energy released - in gigajoules (GJ)	earthquake magnitude	change in magnitude	average rate of change of magnitude
100	4.13	—	—
600	4.65	4.65 - 4.13 = 0.52	$= 0.52/500 = 1.04 \times 10^{-3}$
1100	4.83	0.18	
1600	4.94	0.11	
2100	5.01	0.08	

2) In the table above, calculate the average rate of change of the earthquake magnitude for each interval. Note that the first value has already been calculated for you: 1.04×10^{-3} is the average rate of change of earthquake magnitude as the energy released changes from 100 GJ to 600 GJ. Write your answers in scientific notation.

3) The graph above shows a (dashed) line segment that connects the first two points on the graph. The first point corresponds to an earthquake which releases 100 GJ of energy and the second point corresponds to an earthquake which releases 600 GJ of energy.

Part A: Calculate the slope of this line segment.

Part B: Draw another line segment connecting the second and third point on the graph and calculated its slope.

Part C: How do the slopes of these two lines compare with the average rate of change of earthquake magnitude?

Part D: If you were to draw a third line segment connecting the third and fourth point on the graph what would its slope be?

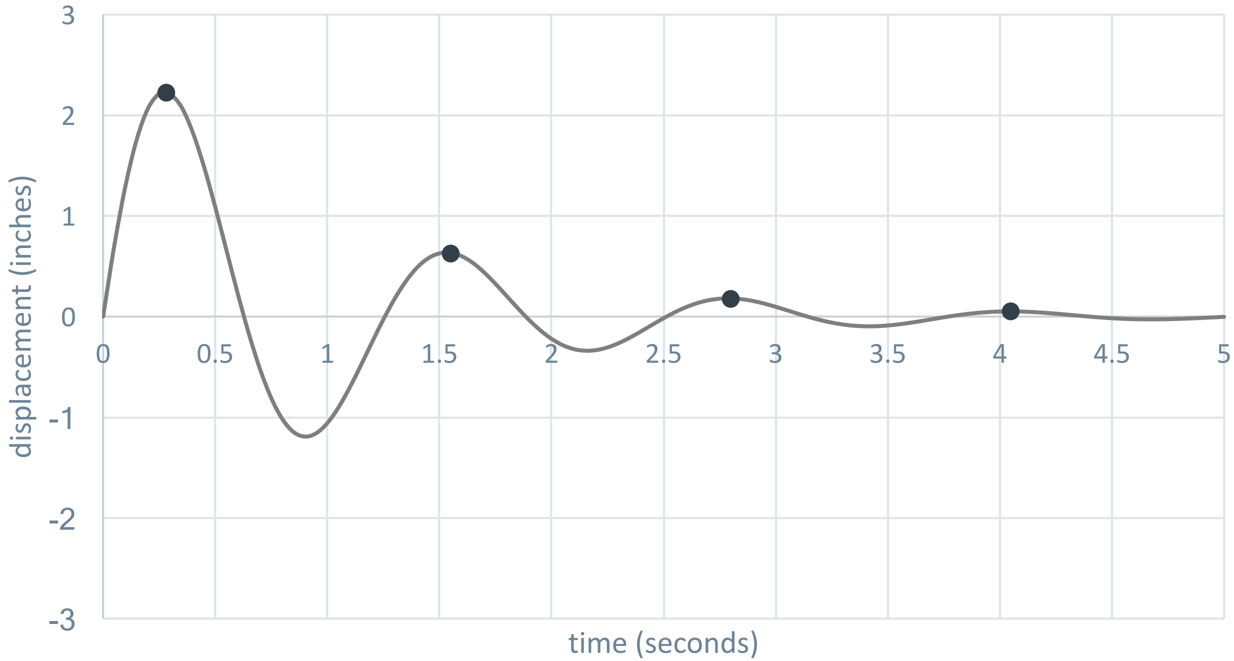
4) Write a sentence that explains how the average rate of change of the magnitude of an earthquake changes as the energy increases in equal amounts.

Engineering

- Late in a course
- Working with exponential functions

Note: Continued work with multiple representations, modeling part of a more complicated function (this function will be revisited later in RF II), “decomposing” a complicated function into manageable pieces.

2) Let's take a look again at the displacement information from the preview assignment. Recall that this data represent the peak displacements of the car t seconds after it hit the speed bump.



time	peak displacements
(seconds)	(inches)
0.30	2.22
1.55	0.622
2.80	0.174
4.05	0.048

Since the peak displacement is an exponential function, we should be able to determine the formula for a function that gives us these displacement values. The formula should have the form:

$$f(t) = Ae^{kt}$$

or

$$f(t) = Ab^{kt}$$

for some base, b, other than e.

Determine a strategy you could use to find the formula for *f(t)*.

3) Find a function of the form $f(t) = Ae^{kt}$ or $f(t) = A\,b^{kt}$ that models the peak displacement data.

Architecture

- How students perceive themselves affects their learning. Help them develop a growth mindset.
- We learn through struggle. Utilize contextualization to create meaningful opportunities for struggle.

Productive Struggle

“The effort to make sense of mathematics, to figure something out that is not immediately apparent.”

Have you tried incorporating productive struggle into your teaching? How did it go?

Productive Struggle

- What do students need to thrive in an environment that incorporates productive struggle?
  - A reason to struggle—Contextualization
  - A community to support their learning
  - A growth mindset—students need to understand and accept that productive struggle is important

How the brain works – psychologically speaking

Growth Mindset

Incremental theory of intelligence

The belief that
academic capabilities
can change with effort

Fixed Mindset

Entity theory of intelligence

The belief that
academic capabilities
are a function of innate ability

How the brain works – psychologically speaking

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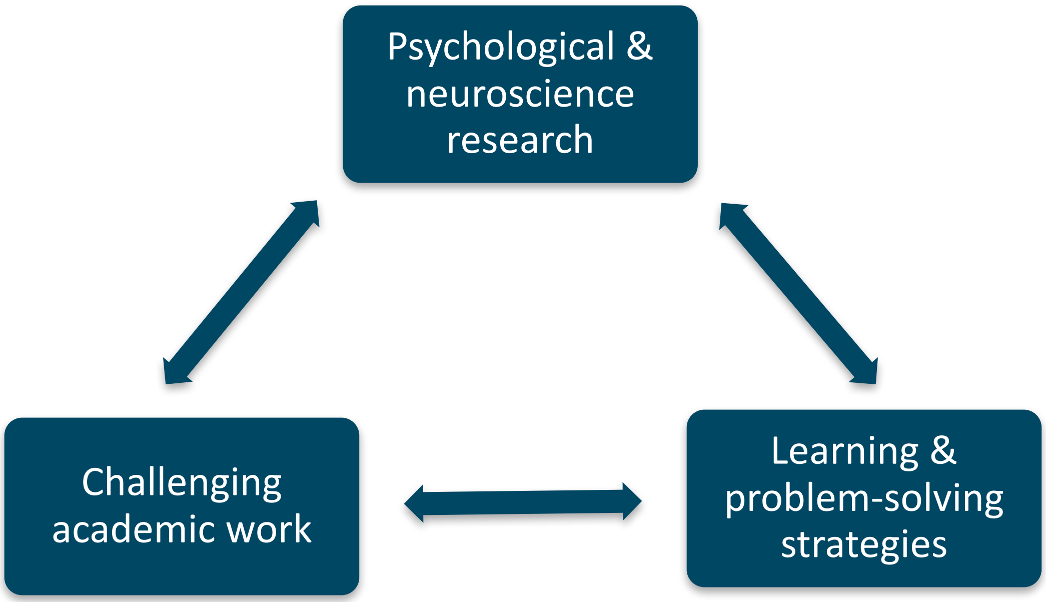
Positive academic behaviors:

- Attending class
- Asking for help
- Enjoying the academic process
- Choosing to tackle challenging tasks

Productive Struggle

- How does incorporating productive struggle change the way we approach teaching?
  - Anticipate student pitfalls and develop facilitating questions that help students persevere in the problem solving process.
  - Leverage collaborative learning to help students persevere
  - Identify and correct misconceptions

A Metacognitive Shift



Classroom culture and climate

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- The DCMP Resource Site
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About the Dana Center

The **Charles A. Dana Center** at The University of Texas at Austin works with our nation's education systems to ensure that every student leaves school prepared for success in postsecondary education and the contemporary workplace.

Our work, based on research and two decades of experience, focuses on K–16 mathematics and science education with an emphasis on strategies for improving student engagement, motivation, persistence, and achievement.

We develop innovative curricula, tools, protocols, and instructional supports and deliver powerful instructional and leadership development.

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