Refreshed!

In this booklet, you'll find the original CRS plus new information on Phase II, the implementaion of the standards.

Washington State

Transition Mathematics Project (TMP)

College Readiness Mathematics Standards

A collaborative project of K-12 schools, community and technical colleges and baccalaureate institutions

TMP is managed by
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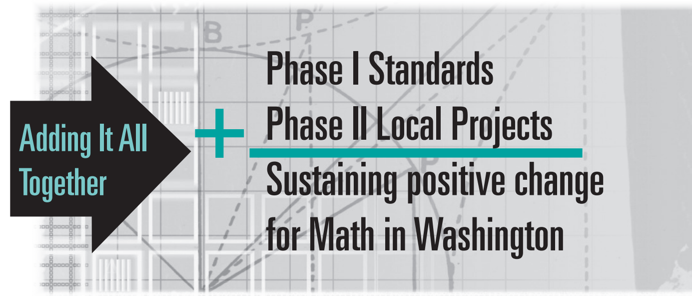
The College Readiness Mathematics Standards



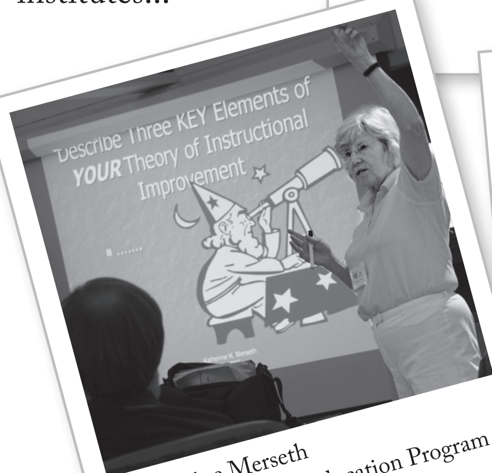
The College Readiness Mathematics Standards define the core knowledge and skills expected of students entering college-level mathematics courses and courses with quantitative components. In addition, they provide the information and support that students need for a successful transition from secondary to postsecondary education in math.

Once the College Readiness Standards were completed, approved and published, the Transition Math Project began Phase II, the implementation stage. With the launch of 18 partnerships across Washington, the TMP is enabling educators to integrate the standards in their classrooms, math departments, and districts, and helping students achieve college readiness in math at the local level.

The main thrust of Phase II was developing and nurturing cross-sector partnerships. Through these partnerships, the TMP helped schools and districts implement the College Readiness Standards into regular classroom instruction. Also during Phase II, the TMP led a process to develop the College Readiness Mathematics Test, based on the College Readiness Standards, to provide 11th and 12th-graders with an early indicator of their preparation levels.

TMP Table of Contents

K-12 teachers and college faculty collaborate at TMP workshops and statewide math institutes...



Katherine Merseth
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Silicon Valley Mathematics Initiative

....in connection with state and national education experts.

You Can Quote Me On That

Washington Mathematics Assessment and Placement (WAMAP)

"Using WAMAP.org made the testing process much more efficient for our staff, and we are all glad the service has been available to us."
– Lisa Wilson, Math Teacher, Renton School District

Whatcom Math Project (WMP)

"Developing and discussing the college readiness standards has given me a clearer understanding of our goals for high school graduates. Knowing where we need to go has helped me think about new ways to reach these goals."

– Brad Dallas, Math Teacher, Nooksack Valley High School

Olympic Peninsula – Transition Math Project (OP–TMP)

"Within my professional learning community, we have made leaps and bounds in our teaching practices over the last three years because of TMP support and professional development."
– Juliana Hochstein, Math Teacher, Bremerton High School

Project TIME

"For two years now, we have had students continue to take math their senior year who most likely would not have if we didn't offer the TMP Project TIME senior math course."
– Deann Anguiano, Math Teacher, Kentridge High School

Calculating Solutions

Math is the greatest barrier preventing students from graduating high school ready for credit-bearing coursework at the college level. Striving to remove that barrier, the Transition Mathematics Project remains well positioned to advance all Washington students' college readiness.

The TMP continues to refine and prioritize the greatest opportunities for sustainable, scalable growth and to identify the most critical remaining gaps between high school preparation and postsecondary success in mathematics.

Through the use of compelling data and proven successes, the TMP has brought an unprecedented sense of urgency, purpose, and focus to the need for improving math performance. Two phases of work were implemented to respond to that need. The success of this work has led to expansions that build on and extend key products and approaches.

Bringing Together All Stakeholders

Inital activities focused on bringing together a diverse cross section of K-12 and higher education stakeholders to develop and build consensus around a common set of college readiness standards. The standards contain the information necessary to plan for and succeed in college-level courses. They encourage every student's highest achievement by defining the knowledge, concepts, and skills they should master. The TMP then distributed over 30,000 copies of these standards across the state.

Implementing the Standards

Additional funding allowed for the development and nurturing of cross-sector regional partnerships to help schools and districts implement the College Readiness Standards. These partnerships included ongoing professional development, events, and innovative products to integrate the standards into daily classroom instruction. The Transition Math Project also led a process to develop a common college readiness math test for early assessment of 11th and 12th-graders. With this dizzying array of math activities throughout the state, TMP relies on enhanced website tools and ongoing development of local partnerships to support teachers.

These tools and approaches include:

- **Web Portals**

Local math projects have yielded statewide advantages. The successful partnerships between the TMP and the 18 Washington schools, districts, and colleges that implemented the College Readiness Mathematics Standards led to the creation of three important new portals on the TMP Website - Curriculum and Instructional Materials, Teacher and Professional Development and Public Outreach. Page 6 of this publication contains links to these portals and other valuable resources.

- **TMP College-School District Partnerships**

This regional partnership phase included an emphasis on capacity building for teachers and schools in addressing the significant challenge of helping all students achieve college readiness in math. Mainly, this has been carried out through local and regional cross-sector partnerships built around the TMP's core principles.

Support of these partnerships has fostered the development of such model products and promising practices as the creation of senior math courses and new sets of math examples and lesson plans. Further, the TMP has acquired the services of Agile Mind, a comprehensive, web-based mathematics instructional and professional development program for teachers, and Academic Youth Development, an algebra readiness program, with the goal of supporting the successful transition of students from middle school to high school mathematics.

Pathways to Future Success

Drawing from its extensive network and in support of promising work around the College Readiness Standards, the TMP is well-positioned to join the nationl focus on developmental math in college with its new Re-thinking Pre-college Math in Washington Colleges program. TMP continues to build on past successes to ensure that all students achieve increased levels of educational attainment in math. The TMP envisions deepening its work in existing partnerships and broadly disseminating lessons learned and practices worthy of attention. TMP's future plans will continue to also fill critical math needs in the transition between high schools and higher education.

Student Attributes

The student attributes section of this document (p. 8) describes a set of student characteristics that the teachers involved in this project believe are essential to being successful in entry college-level classes, particularly math classes. These attributes, including responsibility, perseverance, and attention to detail, reflect attitudes or habits of mind that can lead to better learning; they set the stage for the math-specific standards that follow. Each attribute includes multiple "evidence of achievement" indicators providing examples of specific behaviors that reflect the attribute. Think of the attributes as providing a description of how to learn, while the standards succinctly define what to learn with respect to preparation for college mathematics.

A 2009-2010 Washington state initiative, Student Attributes for Math Success (SAMS), recently received funding from College Spark Washington to deepen and extend existing curricular work and teacher professional development in support of implementing the TMP Student Attributes. High school teachers, college faculty, and administrative leaders comprising five state demonstration sites will lead this effort. New resources, products, and lessons learned from this effort will be displayed on the TMP website *(www.transitionmathproject.org)*.

Standards

The standards section (p. 9-19) offers detailed descriptions of the eight math standards comprising what students need to know and be able to do to be prepared for college-level work in mathematics in Washington state. These standards represent a collective consensus of a wide range of educators from around the state, representing both math and other disciplines involving quantitative reasoning; the key individuals involved in the design and refinement of these standards are listed at the end of the document. Each standard is clearly stated and includes a set of components which represent specific elements within the broader standard. For each component there are evidence-of-learning indicators which clarify specific behaviors or performances that would demonstrate the particular component involved. Although the standards are necessarily presented as separate and distinct areas, they are clearly inter-connected, building upon each other. Taken as a whole, they reflect a continuum that emphasizes both procedural and conceptual mathematical competence.

Extra Expectations

Most of the College Readiness Standards reflected here represent the basic expectations for the variety of entry-level college math (and other disciplines requiring quantitative reasoning) in Washington two- and four-year public institutions. Students needing to take higher level math courses when they enter college, specifically calculus readiness, will need additional skills and knowledge to be prepared for those courses. These extra expectations are embedded throughout the content standards (standards 4 through 8) and are indicated by italicized text to distinguish them from the basic expectations.

Crosswalk—An Intersection of Mathematics Standards

The Crosswalk is a tool to examine the relationship between the Washington State High School Mathematics Standards and the TMP College Readiness Standards.

The Crosswalk is the result of Washington teachers and faculty examining the commonalities and intersections between the Washington State High School Mathematics Standards and the TMP's College Readiness Standards. This new Crosswalk outlines the relationship between these two sets of important state standards. It also includes a key indicating the assessed degree of alignment, along with teacher-faculty comments and suggestions.

For additional information, visit these Web sites:

- **Crosswalk interactive online resource**
  www.transitionmathproject.org/crosswalk
- **TMP College Readiness Standards**
  www.transitionmathproject.org/standards
- **Washington State Mathematics Standards**
  www.k12.wa.us/Curriculuminstruct/mathematics/default.aspx

The Power of Technology

Technological tools such as calculators and computers help students explore the world of mathematics and enrich their mathematics experience. See page 20 for a statement on the appropriate use of technology, representing the collective position of the teachers involved in developing the College Readiness Standards.

About the Transition Math Project

Who We Are

The TMP is a collaborative venture comprising educators from K–12 schools, community and technical colleges, and baccalaureate institutions; staff from the State Board for Community and Technical Colleges, the Superintendent of Public Instruction, the Higher Education Coordinating Board, and the Council of Presidents; and community-based stakeholders and business leaders.

What we have accomplished:

- Completed, approved, and published the College Mathematics Readiness Standards
- Increased the use of math standards and expectations in designing classroom tasks, assignments, and assessments
- Reached more students, especially students underserved by higher education, with clear messages on the importance of math for success after high school
- Developed the College Readiness Math Test to provide a standard definition of college readiness in mathematics
- Developed and disseminated a compendium of workplace tasks that clearly demonstrates how math is used in business and industry contexts

About Phases I & II

Phases I & II set the stage for implementing the College Readiness Mathematics Standards across the state. The first five years of the Transition Mathematics Project laid the groundwork for a seamless system that will benefit all students in Washington public schools. With successful completion of Phases I & II, the TMP has now launched 18 partnerships with local and regional participants. These partnerships have focused on the following key activities:

- Piloting aligned math placement strategies in select regions
- Piloting model curricula in select regions
- Conducting cross-sector classroom teacher-faculty visitations, including team teaching, modeling lessons, and observations
- Incorporating standards-based instructional materials into Senior Math Courses, team-taught by K-12 and college faculty
- Creating and disseminating public outreach materials, and hosting parent-student math information events
- Convening to analyze local outcomes and share best practices

In addition to the initial implementation, Phase II has:

- Shared project results statewide through publications, studies, conference presentations, local institutes and seminars
- Produced model curricula, and distributed statewide
- Made available for pilot-testing a math placement tool aligned to the College Readiness Mathematics Standards and developed by a cross-sector team of faculty and teachers
- Convened best-practices institutes showcasing results and implementation strategies
- Developed policy briefs and shared across the state with local leaders
- Implemented data collection strategies and protocols to mine and interpret data related to project goals to drive instructional change

What is Phase III?

In Phase III, the TMP will deepen and expand the use of its proven resources, tools, and strategies; it will gather specific evidence of the effectiveness of this work in order to target these resources and tools most effectively; and,

- Deepen and significantly expand the local execution of the TMP's most successful work and suite of tools, including the College Readiness Standards
- Strengthen traditional Algebra II classes through improved curriculum and teaching supports
- Offer higher education support for technical assistance to secondary schools struggling in math

At the post-secondary level, the TMP will help two-year colleges with an overhaul of curriculum, assessments, and instructional approaches in pre-college math programs. These Phase III activities will help to ensure student success between remedial math into college-level coursework and toward meaningful goals.

The TMP is positioned to be a low-cost, high-impact option for helping schools and districts successfully implement the Third Year of Math Requirement. Its expertise enables it to explore opportunities to partner with the Washington State Board of Education to develop rich and rigorous CTE courses that provide content through Algebra II.

TMP Local/Regional Partnerships:

- Applied Math Project – Seattle
- Blueprint for Success – Construction Math Initiative
- Columbia Basin College Project
- COMPASS, College and Careers
- Edmonds Articulation Council Project
- Issaquah School District Math/CTE Connections
- North King County Project
- Olympic Peninsula Transition Math Project
- Pierce County Careers Connection Project
- Project TIME
- Sammamish River Access Math Project
- Seattle Transition Project
- Spokane Area Curriculum Collaboration
- Southwest Washington – Transitions in Math Education
- Washington Mathematics Assessment and Placement (wamap.org)
- Walla Walla Area Project
- Whatcom Math Project
- Yakima Valley Project

Divide and conquer: We live in a mathematical world

Much more than just numbers, math is a way of understanding the world that works into everything, from family medicine to firefighting, car mechanics to culinary arts.

Web Sites

Transition Mathematics Project Home Page
www.transitionmathproject.org

College Readiness Standards
www.transitionmathproject.org/standards
A complete .PDF version of this document can be viewed and printed.

TMP Local/Regional Partnerships
www.transitionmathproject.org/partners
TMP-supported local partnership projects in Washington state.

Marketing Toolkit
www.transitionmathproject.org/resources/toolkit.asp
A repository of handouts, fliers, articles, reports, and other marketing and communication tools to support math outreach efforts.

Math Lab
www.transitionmathproject.org/resources/mathlab.asp
A student and parent portal with up-to-date math planning resources, tips, and tools.

Curriculum and Instructional Materials
www.transitionmathproject.org/curriculum
Content-rich products and methods to improve math instruction.

Teacher Professional Development
www.transitionmathproject.org/pro-development
Web resources, rich math examples/tasks and assessments, studies and resource experts to support classroom teaching.

Public Outreach
www.transitionmathproject.org/outreach
Math messaging tools and resources designed to be used in classrooms, conferences, school events, and public meetings.

The Value of Process and Content Standards

The logic inherent in the study of mathematics allows for applications to a broad range of situations in which answers to practical problems can be found with accuracy, as you will see in the workplace examples on this page. The ability to form logical conclusions is a cornerstone of success and requires perceiving interconnectedness. As students progress in the study of math, they learn to:

- Distinguish between inductive and deductive reasoning
- Understand the meaning of logical implication
- Test general assumptions
- Distinguish between something being proven and a mere plausibility argument
- Identify logical errors in chains of reasoning

The College Readiness Standards include process and content standards. Both types of standards are necessary for students to become mathematically proficient.

Process Standards

Process standards describe productive ways of approaching mathematics that apply across many content areas; they are a way of acquiring and using content knowledge.

1. Reasoning / Problem-Solving

Use logical reasoning and math to define and solve problems.

Workplace example:
A veterinarian uses reasoning to calculate dosages of medicines for animals.

2. Communication

Understand and discuss mathematical information in both mathematical and everyday language.

Workplace example:
A salesperson calculates and describes the total cost of shipping an order based on volume, weight, and distance shipped.

3. Connections

Use math and mathematical thinking in other subjects.

Workplace example:
A carpenter counts the number of days booked for each month to calculate the amount of time available for new jobs.

Content Standards

The understanding of mathematical concepts has become imperative for every citizen as everyday functions become more mathematically complex. Content standards are what we normally think of when someone talks about math.

4. Number Sense

Apply math to a situation and be able to explain how you reached your correct conclusion.

Workplace example:
An interior designer uses number sense to figure how much fabric and the cost to make custom bedroom curtains.

5. Geometry

Use geometry to back up a claim or observation, model situations, and draw conclusions.

Workplace example:
A plumber uses geometry to calculate the total length of pipe needed for a job, including corners and curves.

6. Probability / Statistics

Analyze data using probability and statistics.

Workplace example:
A baseball coach uses pitching and batting statistics to set his lineup and pitching rotation.

7. Algebra

Use algebra to solve equations and model complex situations.

Workplace example:
An excavator uses algebra to accurately remove the required cubic yardage of earth to make space for a building foundation.

8. Functions

Use function concepts and procedures to understand mathematical relationships.

Workplace example:
A loan officer calculates the total amount of interest on a loan, depending on the amount the buyer pays at the time of purchase.

Together, the standards describe the basic skills and understanding students will need in order to function effectively in the 21st century.

Extra Expectations

See page 4 for information about Extra Expectations for students who will take calculus courses when they enter college.

Student Attributes

Success in college depends on a student's ability to respond to the challenges presented by new problems and new ideas. In addition to the process and content standards that follow, the attributes described below are crucial to success in college-level courses, both in mathematics and in other disciplines.

Attributes	Evidence of Achievement
■ Demonstrates intellectual engagement.	■ Perceives mathematics as a way of understanding—a view that mathematics must make sense and is not just a sequence of algorithms to be memorized and applied. ■ Actively explores new ideas, posing questions about their meaning, significance, and implications. ■ Recognizes patterns—as well as deviations—from previously learned patterns in data, diagrams, symbols, and words. ■ Understands that abstraction and generalization are important sources of the power of mathematics. ■ Takes risks and is willing to be challenged as part of the learning process. ■ Contributes to and benefits from group problem-solving activities.
■ Takes responsibility for own learning.	■ Attends nearly every class session and when absent, seeks ways to learn the material covered in class. ■ Conscientiously prepares work assigned for class. ■ Examines and learns from his or her errors to clarify understanding. ■ Takes advantage of available resources when needed—class time, notes, textbook, assignments, tutoring services, supplemental materials, study groups, etc. ■ Sets aside as necessary time to be successful.
■ Perseveres when faced with time-consuming or complex tasks.	■ Works on problems that require time and thought, particularly problems that cannot be solved by mimicking a previously seen example. ■ Organizes and implements multiple steps, concepts, or techniques in order to complete tasks. ■ Recognizes when an approach is unproductive and makes logical modifications to that approach or switches to another approach. ■ Is convinced that effort is an important component of success in mathematics.
■ Pays attention to detail.	■ Correctly follows all parts of oral and written directions without needing additional reminders. ■ Makes few notational errors, e.g., accidentally changing digits, dropping or altering algebra symbols, incorrectly positioning points on a grid, etc.

Reasoning/Problem-Solving

The student uses logical reasoning and mathematical knowledge to define and solve problems.

Components	Evidence of Learning
1.1 Analyze a situation and describe the problem(s) to be solved.	a. Extract necessary facts and relationships from the given information. b. Identify and supply additional information needed to solve each problem.
1.2 Formulate a plan for solving the problem.	a. Evaluate the advantages and disadvantages of different strategies, representations, and tools (including various forms of technology) for solving the problem. b. Choose concepts, strategies, representations, models, and tools well-suited to solving the problem.
1.3 Use logical reasoning and mathematical knowledge to obtain and justify correct solutions.	a. Correctly execute a plan to solve the problem. b. Evaluate and revise the solution method when it appears unlikely to produce a reasonable or suitably accurate result. c. Evaluate potential solutions for appropriateness, accuracy, and suitability to the context of the original problem. d. Provide oral, written, and/or symbolic explanations of the reasoning used to obtain a solution. e. Make and justify a multi-step mathematical argument providing appropriate evidence at each step. f. Use a variety of approaches—inductive and deductive, estimations, generalizations, formal, and/or informal methods of proof—to justify solutions.

Communication

The student can interpret and communicate mathematical knowledge and relationships in both mathematical and everyday language.

Components	Evidence of Learning
2.1 Summarize and interpret mathematical information which may be in oral or written formats.	a. Summarize and interpret many different types of graphs. b. Recognize and explain the meaning of information presented using mathematical notation. c. Create symbolic representations for situations described in everyday language.
2.2 Use symbols, diagrams, graphs, and words to clearly communicate mathematical ideas, reasoning, and their implications.	a. Identify the variables and constants used. b. Identify units associated with these variables and constants. c. Use correct mathematical symbols, terminology, and notation.
2.3 Produce mathematically valid oral, written, and/or symbolic arguments to support a position or conclusion, using both mathematical and everyday language.	a. Create explanations that are appropriate to the needs of the audience and the situation. b. Use appropriate details or evidence to support the explanation.

Connections

The student extends mathematical thinking across mathematical content areas and to other disciplines and real life situations.

Components	Evidence of Learning
3.1 Use mathematical ideas and strategies to analyze relationships within mathematics and in other disciplines and real life situations.	a. Compare and contrast the different mathematical concepts and procedures that could be used to complete a particular task. b. Recognize patterns and apply mathematical concepts and procedures in other subject areas and real world situations.
3.2 Understand the importance of mathematics as a language.	a. Connect mathematical definitions and procedures with underlying math concepts. b. Transfer mathematical vocabulary, concepts, and procedures to other disciplinary contexts and the real world. c. Construct procedures and concepts from mathematical definitions.
3.3 Make connections by using multiple representations, e.g., analytic, numeric, and geometric.	a. Integrate mathematical content areas by using multiple representations. b. Use multiple representations to demonstrate understanding of links between math and other disciplines, and real world situations.
3.4 Abstract mathematical models from word problems, geometric problems, and applications.	a. Recognize and clarify mathematical structures that are embedded in other contexts. b. Describe geometric objects and shapes algebraically. c. Compare and contrast different mathematical models.

Number Sense

The student accurately describes and applies concepts and procedures related to real and complex numbers.

Components	Evidence of Learning
4.1 Understand the concept of real numbers.	a. Explain the meaning of the square root of a number, including why negative numbers have no real square roots. b. Describe a situation that requires an irrational number and provide an example of an irrational number. c. Apply the definition for negative integer exponents of real numbers. d. Represent real numbers with rational exponents. e. Compare and order real numbers without a calculator using relationships between integers, and the effects of radicals and rational exponents on those relationships.
4.2 Accurately and efficiently compute with real numbers in all forms, including rational exponents and scientific notation.	a. Complete multi-step computations using the order of operations and the properties of operations (associative, commutative, distributive, etc.) in situations involving combinations of real numbers. b. Recognize and justify the need for an exact answer in a given situation. c. Select and justify appropriate strategies and tools to compute the answer in a problem.
4.3 Apply estimation strategies using real numbers.	a. Recognize and justify the sufficiency of estimating or approximating the result of a calculation involving real numbers. b. Use estimation to predict or verify the reasonableness of calculated results.
4.4 Understand the concept of complex numbers and perform computations with complex numbers.	*a. Add, subtract, multiply, and divide complex numbers; express in standard form and explain the properties (i.e., why useful; when used).* *b. Describe a situation that requires a complex number (include an algebraic example and a graphical example).*

Note: For an alignment between the recently revised Washington State Mathematics Standards and the College Readiness Standards, refer to the Crosswalk at: *www.transitionmathproject.org/crosswalk*

italic text=extra expectations

Geometry

The student makes hypotheses, models situations, draws conclusions, and supports claims using geometric concepts and procedures.

Components	Evidence of Learning
5.1 Make and test conjectures about 2-D figures (polygons and circles) and 3-D figures (spheres, right prisms and pyramids, right circular cylinders and cones), or figures constructed from these shapes.	a. Use physical, symbolic, and technological models to explore conjectures.
5.2 Represent the relevant features of a physical situation using 2-D figures with and without a coordinate system	a. Use basic 2-D figures such as circles or polygons to represent objects essential to a situation. b. Include additional line segments to represent important known or unknown distances. c. Introduce a coordinate system when useful for describing the position of objects in a situation.
5.3 Use properties of and relationships between 2-D or 3-D figures to draw and justify conclusions about a situation represented with such figures with or without a coordinate system.	a. Inductively generate a conjecture and deductively support it. b. Apply and justify the applicability of transformations, congruence, similarity, ratios, and proportions in problem-solving situations. c. Distinguish between area and perimeter of 2-D figures, surface area, and volume of 3-D figures. d. Calculate the area and perimeter of circles, triangles, quadrilaterals, and regular polygons. e. Use the Pythagorean Theorem (or distance formula) in 2-D and 3-D situations when appropriate to compute unknown distances. f. Calculate the volume and surface area of spheres, right rectangular prisms, and right circular cylinders, *right prisms, right pyramids, and right cones.* *g. Graph ellipses and hyperbolas whose axes are parallel to the x and y axes, and demonstrate understanding of the relationship between their standard algebraic form and their graphical characteristics.*
5.4 Recognize and apply the basic right-triangle trigonometric relationships (sine, cosine, and tangent) to solve problems.	a. Use sine, cosine, or tangent to find unknown distances *and angles.* b. Use the inverse of sine, cosine, or tangent to find the measure of a missing angle. *c. Understand the relationship of cotangent, secant, and cosecant to basic right-triangle ratios.* *d. Use Law of Cosines and Law of Sines to solve problems.*

italic text=extra expectations

Note: For an alignment between the recently revised Washington State Mathematics Standards and the College Readiness Standards, refer to the Crosswalk at: *www.transitionmathproject.org/crosswalk*

Probability/Statistics

The student accurately describes and applies concepts and procedures from probability and statistics to analyze data.

Components	Evidence of Learning
6.1 Use empirical/experimental and theoretical probability to investigate, represent, solve, and interpret the solutions to problems involving uncertainty (probability) or counting techniques.	a. Describe and apply the concepts of complementary, mutually exclusive, independent, and compound events. b. Describe and apply procedures for computing and comparing theoretical probabilities and empirical/experimental results. c. Describe and apply procedures for counting techniques such as the Fundamental Counting Principle, permutations, and combinations.
6.2 Develop informative tables, plots, and graphic displays to accurately represent and study data.	a. Use and interpret pie charts, bar graphs, histograms, box-and-whisker plots, scatter plots, stem and leaf, and line graphs. b. Analyze data displays to evaluate the reasonableness of claims, reports, studies, and conclusions. c. Justify the use of appropriate graphical displays to accurately represent and study data. d. Determine trends, predicted values and possible causes of skewed and clustered distributions.
6.3 Develop and evaluate inferences and predictions that are based on data.	a. Use measures of central tendency (mean, median, mode) and spread (range, quartiles) to summarize data, draw inferences, make predictions, and justify conclusions. b. Develop and conduct an investigation drawing appropriate conclusions through the use of statistical measures of center, frequency, and spread, combined with graphical displays.
6.4 Create and evaluate the suitability of linear models for a data set.	a. Create, select, and justify an appropriate linear model for a given set of data. b. Use reasonable models to make predictions and justify conclusions. c. Recognize when arguments based on data confuse correlation with causation.

Note: For an alignment between the recently revised Washington State Mathematics Standards and the College Readiness Standards, refer to the Crosswalk at: *www.transitionmathproject.org/crosswalk*

Algebra

The student accurately describes and applies concepts and procedures from algebra.

Components	Evidence of Learning
7.1 Recognize and use appropriate concepts, procedures, definitions, and properties to simplify expressions and solve equations.	a. Explain the distinction between factor and term. b. Explain the distinction between expression and equation. c. Explain the distinction between simplify and solve. d. Know what it means to have a solution to an equation. e. Use properties of equality to solve an equation through a series of equivalent equations. f. Use appropriate properties to simplify an expression, resulting in an equivalent expression. g. Recognize the equivalence between expressions with rational exponents and radicals.
7.2 Combine and simplify algebraic expressions that contain polynomials, rational expressions, radicals, or rational exponents. *7.2 Combine and simplify algebraic expressions that contain polynomials, rational expressions, radicals, positive or negative rational exponents, and logarithmic expressions.*	a. Find the sum, difference, or product of two polynomials, then simplify the result. b. Factor out the greatest common factor from polynomials of any degree *and from expressions involving rational exponents.* c. Factor quadratic polynomials with integer coefficients into a product of linear terms. d. Simplify quotients of polynomials given in factored form, or in a form which can be factored *or determine if irreducible over the real numbers.* e. Add, subtract, multiply, and divide two rational expressions of the form, $\frac{a}{bx+c}$ where a, b, and c are real numbers such that $bx+c \neq 0$ *and of the form* $\frac{p(x)}{q(x)}$, *where p(x) and q(x) are polynomials.* f. Simplify products and quotients of single-term expressions with rational exponents (rationalizing denominators not necessary). *g. Simplify products and quotients of expressions with rational exponents and rationalize denominator when necessary.* *h. Simplify rational expressions that involve complex fractions.* *i. Simplify logarithmic expressions.* *j. Factor polynomials over the complex numbers, if possible, and relate to the Fundamental Theorem of Algebra.*

italic text=extra expectations

Note: For an alignment between the recently revised Washington State Mathematics Standards and the College Readiness Standards, refer to the Crosswalk at: *www.transitionmathproject.org/crosswalk*

italic text=extra expectations

Components	Evidence of Learning
7.3 Solve various types of equations and inequalities numerically, graphically, and algebraically; interpret solutions algebraically and in the context of the problem; distinguish between exact and approximate answers.	a. Solve linear equations in one variable. b. Solve linear inequalities in one variable, including those involving "and" and "or." c. Solve systems of linear *and nonlinear* equations in two variables. d. Solve linear inequalities in two variables (graphically only) *and nonlinear inequalities—numerically, graphically and algebraically.* e. Solve absolute value equations of the form $\|ax + b\| = c$. f. Use a variety of strategies to solve quadratic equations including those with irrational solutions and recognize when solutions are non-real. *Simplify complex solutions and check algebraically. Solve quadratic equations by completing the square and by taking roots.* g. Solve equations in one variable containing a single radical *or two radicals.* h. Solve exponential equations in one variable (numerically, graphically, *and algebraically).* i. Solve rational equations in one variable that can be transformed into an equivalent linear or quadratic equation (limited to monomial or binomial denominators). j. Solve literal equations (formulas) for a particular variable. *k. Solve logarithmic equations.* *l. Solve rational equations and inequalities with polynomial denominators.* *m. Solve absolute value and polynomial inequalities and justify solution.*
7.4 Demonstrate an understanding of matrices and their applications	*a. Add, subtract and multiply 2x2 matrices.* *b. Find the inverse of a 2x2 matrix.* *c. Evaluate the determinant of 2x2 and 3x3 matrices.* *d. Solve 2x2 and 3x3 systems of linear equations using matrices or the determinant.*
7.5 Demonstrate an understanding of sequences and series.	*a. Identify a sequence as arithmetic or geometric, write an expression for the general term, and evaluate the sum of a series based upon the sequence.* *b. Construct terms of a series from the general formula and use summation notation.* *c. Use induction to prove theorems about sums of series.*

Functions

The student accurately describes and applies function concepts and procedures to understand mathematical relationships.

italic text=extra expectations

Components	Evidence of Learning
8.1 Recognize functional relationships presented in words, tables, graphs, and symbols.	a. Recognize whether a relationship given in a symbolic, graphical, or tabular form is a function. b. Determine the domain *and range* of a function. c. Understand and interpret function notation, particularly as it relates to graphic displays of data. *d. Demonstrate an understanding of parametric equations.*
8.2 Represent basic functions (linear, quadratic, exponential, and reciprocal) and piecewise-defined functions (varying over sub-intervals of the domain) using and translating among words, tables, graphs, and symbols. *8.2 Represent basic functions listed above, piecewise-defined functions (varying over subintervals of the domain), and the following advanced functions (cubic, quartic, logarithmic, square root, cube root, absolute value, and rational functions of the type* $f(x)=\frac{1}{x-a}$*) using and translating among words, tables, graphs, and symbols.*	a. Evaluate functions to generate a graph. b. Describe relationships between the algebraic features of a function and the features of its graph and/or its tabular representation. c. Use simple transformations (horizontal and vertical shifts, reflections about axes, *shrinks and stretches)* to create the graphs of new functions using linear, quadratic, and/or absolute value functions. *cubic, quartic, exponential, logarithmic, square root, cube root, absolute value, piecewise, and rational functions of the type* $f(x)=\frac{1}{x-a}$. d. Algebraically construct new functions using addition and subtraction (e.g., profit function), *multiplication, division, and composition.* *e. Given an algebraic representation of a rational function, find the intercepts, asymptotes (horizontal, vertical, and slant), and holes (discontinuities), then sketch the graph.* *f. Given a graph or graphical features, including degrees, intercepts, asymptotes, and/or holes (discontinuities), generate an algebraic representation of a polynomial or rational function.* *g. Sketch the graph of a polynomial given the degree, zeros, max/min values, and/or initial conditions.* *h. Graphically/numerically construct new functions using addition, subtraction, and composition.* *i. Identify the components of composite functions (e.g., given* $f \circ g \circ h$*, find suitable functions f, g, and h) and determine the domain and range.*

Note: For an alignment between the recently revised Washington State Mathematics Standards and the College Readiness Standards, refer to the Crosswalk at: *www.transitionmathproject.org/crosswalk*

Functions (continued)

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Components	Evidence of Learning
8.3 Analyze and interpret features of a function.	a. Describe patterns in the function's rate of change, identifying intervals of increase, decrease, constancy, and, if possible, relate them to the function's description in words or graphically (using graphic calculator). b. Identify y-intercepts and zeros using symbols, graphs, and tables. c. Identify extrema and trends using graphs and tables. *d. Recognize and sketch, without the use of technology, the graphs of the following families of functions: linear, quadratic, cubic, quartic, exponential, logarithmic, square root, cube root, absolute value, and rational functions of the type* $f(x)=\frac{1}{x-a}$, *using the symmetry of odd and even functions when appropriate.* *e. Understand the relationship between the degree of a polynomial and the number of roots; interpret the multiplicity of roots graphically.*
8.4 Model situations and relationships using a variety of basic functions (linear, quadratic, *logarithmic,* exponential, and reciprocal) and piecewise-defined functions.	a. Choose a function suitable for modeling a real-world situation presented using words or data. b. Determine and interpret the meaning of rates of change, intercepts, zeros, extrema, and trends. c. Abstract mathematical models from word problems and interpret solutions in the context of these source problems. d. Identify and justify whether a result obtained from a function model has real world significance.
8.5 Recognize, analyze, and interpret inverse functions.	*a. Explain the conceptual meaning of inverse functions using graphs, tables, in words, and arrow diagrams.* *b. Define what it means for a function to be one-to-one, identify examples and non-examples (algebraic and graphical), and generate examples (algebraic and graphical).* *c. Find and verify the inverse function algebraically, graphically, and numerically; restrict the domain of a function when necessary.*

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Components	Evidence of Learning
8.6 Recognize, analyze, interpret, and model with trigonometric functions.	*a. Represent and interpret trig functions using the unit circle.* *b. Demonstrate an understanding of radians and degrees by converting between units, finding areas of sectors, and determining arc lengths of circles.* *c. Find exact values (without technology) of sine, cosine and tangent for unit circle and for multiples of $\pi/6$ and $\pi/4$; evaluate trigonometric ratios; and distinguish between exact and approximate values when evaluating trig ratios/functions.* *d. Sketch graphs of sine, cosine, and tangent functions, without technology; identify the domain, range, intercepts, and asymptotes.* *e. Use transformations (horizontal and vertical shifts, reflections about axes, period and amplitude changes) to create new trig functions (algebraic, tabular, and graphical).* *f. Know and apply the identity $\cos^2 x + \sin^2 x = 1$ and generate related identities; apply sum and half-angle identities.* *g. Solve linear and quadratic equations involving trig functions.* *h. Generate algebraic and graphical representation of inverse trig functions (arcsin, arccos, arctan), and determine domain and range.* *i. Use trig and inverse trig functions to solve application problems.*

Technology Statement

Technology (e.g., calculators and computers) provides invaluable tools to help students explore the world of mathematics and enrich their mathematics experience. Technology helps students organize data and solve complex problems, and it facilitates a broader understanding of data and problem-solving by providing tools of analysis and visualization. For example, in an algebraic setting, graphing calculators can maximize class time, allowing easy movements between different representations of functions.

Running simulations with multiple trials, checking graphic solutions, and testing the behavior of functions with different parameters are all accessible with this technology. Instead of just memorizing rules and definitions in geometry, students perform constructions, measure figures, observe patterns, discuss their findings, write their own definitions, and formulate their own geometric conjectures with the use of technology.

At the same time, it is vital that students develop foundational mathematics skills that allow them to appreciate and use technology appropriately. The deep understanding of mathematics that is a by-product of learning these foundational skills, including a mastery of basic algorithms and graphing procedures, helps students make good decisions about the appropriate use of technology and its functionality in problem solving settings. Development of this fundamental understanding is central to producing students who are capable of handling college-level work in mathematics without needing remediation on entry.

The College Readiness Standards neither depend on nor reflect a particular curriculum or specific set of courses; we believe that there are multiple ways that the competencies reflected in the standards can be achieved. We would suggest, however, that a sound preparatory curriculum will involve both:

1. Developing foundational skills, including basic algorithms and graphing procedures, plus a deep understanding of mathematical concepts, and
2. Ensuring that students are capable of using technology to help them
   - solve problems that legitimately need technology
   - explore mathematical concepts.

Proficiency with technology is not equivalent to competency in mathematics. Students need to be successful in meeting *both* objectives but with the awareness that technology is a means to strengthening mathematical understanding and not a substitute for it. At the same time, it is also crucial that all students have access to and experience with appropriate technology tools (e.g., graphing calculators) that can be useful in doing the kind of mathematical problem solving and analysis fundamental to many college-level math courses.

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www.sbctc.ctc.edu

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Council of Presidents representing Washington's public baccalaureate institutions
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www.hecb.wa.gov

1/10 P2 V.1